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NONLINEAR PHYSICAL CHARACTERISTICS OF A HIGH-VOLTAGE ELECTRIC DISCHARGE IN A TWO-PHASE LIQUID

K. Kusaiynov and S. E. Sakipova

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Physical features of an electric discharge in a gas-liquid medium are investigated; a direct relationship between the entropy change and the fractal nature of the medium itself is shown. Simplified equations and laws of conservation that describe the dynamics of the electric discharge in the two-phase liquid with allowance for the thermodynamic properties of the medium as functions of its fractal structure are used.

Pulsed pressure of an explosion that occurs in a high-voltage electric discharge in water is widely used in technology for leaking, shaping, and working of materials. A complete description of the phenomenon in question calls for a complex of investigations of electric and thermodynamic properties of a partically ionized plasma and a gas-vapor medium, the dynamics of shock waves formed in a compressible medium, and their interaction with a solid surface. Among the wide scope of problems, we can recognize the study of physical features of an electrohydraulic effect in a multiphase medium that has received a great deal of attention recently [1-4]. This is due to the fact that a high-power explosion is necessarily accompanied by phase changes in the medium. Besides, we should take into account that, in practice, industrial water always contains bubbles of air (impurities) that do not practically alter the density of the liquid but lead to a strong change in the compressibility of the liquid and hence to the entire dynamics of the electroexplosion. Physical phenomena in a multiphase medium are always static in character. In the case of a two-phase liquid, the aforesaid manifests itself as strong nonlinearity – the turbulence and the fractal (structural) nature of the phenomenon. In the work, we also proposed an approach for allowing for the effect of the nonlinearity of the medium on its electrical characteristics.

1. Physical Features of an Electric Discharge in a Two-Phase Liquid. Electric discharges whose characteristic time is evaluated as $t = 2\pi\sqrt{LC} \sim 10^{-4}-10^{-6}$ sec are used for technical purposes. In this time interval, the working medium of macroscopic sizes has no time to exchange heat with the ambient medium and because of this one condition of adiabaticity of the process of electroexplosion is satisfied. The condition of equilibrium (isoentropy) should also be satisfied, which requires slowness of the process relative to the relaxation properties of the medium. This condition is not satisfied in electroexplosion, which becomes evident from a simple evaluation of the quantities. The characteristic relaxation time for an electronic plasma generated in electroexplosion is evaluated as the ratio of the Debye radius to the thermal rate

$$\tau_r \sim r_D / v_t \sim (m/4\pi ne^2)^{1/2}, \quad (1)$$

where m, n, and e are the mass, concentration, and charge of the electrons. In plants commonly used in practice, the pressure is $P \sim 10^8-10^9$ N/m^2 and the temperature is $T \sim 10^4$ K. Taking into account that $P \sim nkT$ we have $\tau_r \sim 10^{-4}$ sec, i.e., the equilibrium condition is not satisfied. The entropy changes due to the structural rearrangement of the working medium. In the case of a two-phase liquid, because of the larger compressibility of the medium the perturbation velocity (the velocity of sound) will turn out to be smaller than the hydrodynamic velocity, and electroexplosion will necessarily be accompanied by the formation of shock waves, on the front of which all the hydrodynamic characteristics experience a jump. For this reason, use of an empirical equation of state

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in the Thate form [1], which holds true for an isoentropic process in pure water, is not legitimate. We use the equation of a polytropic process

$$PV^{\kappa} = \text{const}, \tag{2}$$

where κ is a polytropic exponent dependent on the structure of the medium. The necessity of using a polytropic equation is also pointed to in [2], where the range of experimental values of κ for a real gas (water vapor) is given

$$\kappa = 1.01 - 2.7. \tag{3}$$

If the entropy changes only due to the spatial rearrangement of the medium (because of the absence of heat transfer there is no dependence on pressure and temperature) without allowance for the interaction of macroscopic structure, i.e., the nonideality of the medium, we can resort to the relation

$$\kappa = (i + 2)/i, \tag{4}$$

where i is the degree of freedom of the medium [5]. In the problem in question, i is the degree of freedom of motion of the macroscopic structural elements formed in a nonlinear nonequilibrium medium in pulsed energy supply rather than of a molecular level of motion. An example of structural motion is hydrodynamic turbulence [6], where i is governed by the number of individual vortices of the liquid. In developed turbulence, the number of macroscopic degrees of freedom [7] is

$$i \sim (r_m/r_0)^3 \sim (\mathrm{Re}/\mathrm{Re}_{cr})^{9/4}, \tag{5}$$

where r_0 and r_m are the space scales of the motion. In the problem in question, it is not convenient to use formula (5) since the arbitrariness exists in finding Re_{cr}. It is necessary to select a more universal method for determination of i.

The most specific properties of the structural nature of a medium, whatever its concrete character, are fractal characteristics. The objects of a structural and self-similar composition are called fractals [8]. We can relate the fractal dimensionality by the scaling index γ that characterizes spatial-invariant deformations to the coefficient of gas content φ in a two-phase liquid. Let r_0 and r_m be the scales of the structures of gaseous-phase bubbles in the two-phase medium. Then the corresponding fractal $S(\gamma)$ and smooth S surfaces are determined by the relation

$$S(\gamma)/S = (r_m/r_0)^{\gamma}. \tag{6}$$

In [10, 11], an example of calculation of γ for concrete laws of fractalization of a medium is given where the limiting values $\gamma_* = 0.7925$ and $\gamma_\perp = 0.465$ that correspond to isotropic and anisotropic cases are found. All possible forms of perturbations and deformations are shown to be found between them. In view of (6), for a fractal of spherical symmetry we have

$$S(\gamma) = 4\pi a_{\mathrm{eff}}^2, \quad a_{\mathrm{eff}} = a(r_m/r_0)^{\gamma/2}, \quad V(\gamma) = 4\pi a_{\mathrm{eff}}^3/3,$$

$$i = V(\gamma)/V = (r_m/r_0)^{3\gamma/2}. \tag{7}$$

We relate the scaling index γ to the volumetric gas content φ. The relative volume of the liquid phase is determined by

$$V_{\mathrm{liq}}/(V_{\mathrm{liq}} + V_g) = 1 - \varphi, \tag{8}$$

where V_{liq} and V_g are the volumes of the liquid and gaseous phases. Using (6), in cylndrical and spherical cases, respectively, we have:

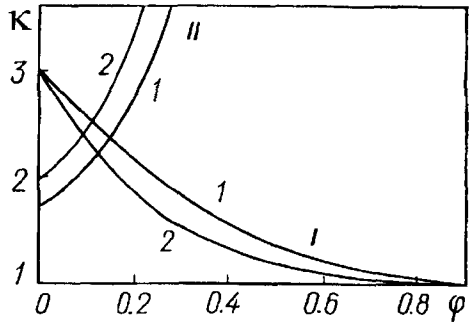

Fig. 1. Polytropic exponent vs. average (*I*) and local (*II*) gas contents of cylindrical (1) and spherical (2) volumes.

$$\langle r_m\rangle/r_0 = (1-\varphi)^{-1/\gamma} \quad \text{and} \quad \langle r_m\rangle/r_0 = (1-\varphi)^{-2/3\gamma}. \tag{9}$$

Formulas (9) determine $\langle r_m\rangle/r_0$ of the gaseous phase in terms of the average value of φ.

For a local gas content φ, we need to find the local scale of the fractal structures r_m/r_0 of the liquid phase. Calculating by integration the volume of liquid drops with a fractal surface, we obtain, respectively, for cylindrical and spherical cases:

$$\frac{r_m}{r_0} = \frac{2+\gamma}{2}(1-\varphi)^{1/\gamma} \quad \text{and} \quad \frac{r_m}{r_0} = \frac{3+\gamma}{3}(1-\varphi)^{1/\gamma}. \tag{10}$$

In (10), the limiting transition $\varphi \to 0$ corresponds to the limit $\gamma \to 0$; the topological dimensions of the object itself are $d = 3$ in spherical and $d = 2$ in cylindrical cases (consideration is given to the fractal surface of the liquid phase). In the case of a bubble state of the medium (for an averaged space scale), the topological dimensions of the space into which the object in question is placed are taken for convenience as d; therefore, in formulas (9) the limit $\varphi \to 0$ corresponds formally to $\gamma \to 1$.

In an electric discharge in a multiphase liquid, there form complex interacting fractals – multifractals that have a set of fractal dimensions. The dimensions of a first-order multifractal govern the entropy of the medium [8]. Thus, the entropy change in electroexplosion in a two-phase medium is closely connected with the fractal nature of the latter. The cavity of the discharge is basically filled with a vapor-liquid component [2] and partially with a plasma. If necessary, the expression $(r_m/r_0)^\gamma$ can also be found for the plasma. For this purpose, γ is determined separately in terms of the multifractal dimensions [8], while r_m is determined in terms of the Debye radius of a multicomponent plasma.

The degree of freedom of macroscopic motions takes on practically all the continuous values in the interval $(1 \le i \le \infty)$, while for a molecular level of motions, i are limited and discrete (for example, for monatomic molecules, $i = 3$, for diatomic molecules, $i = 5$). Consequently, from (4) we obtain the interval $1 \le \kappa \le 3$, which is similar to the experimental range of κ determined according to (3). From relations (4), (7), and (9), we obtain the polytropic exponent as a function of φ and γ for the averaged fractal scale:

$$\kappa = [2 + (1-\varphi)^{-2/\gamma}](1-\varphi)^{2/\gamma} \quad \text{and} \quad \kappa = [2 + (1-\varphi)^{-3/\gamma}](1-\varphi)^{3/\gamma} \tag{11}$$

and for the local fractal scale:

$$\kappa = \left\{2 + [(2+\gamma)/2]^3 (1-\varphi)^{3/\gamma}\right\} [2/(2+\gamma)]^3 (1-\varphi)^{-3/\gamma}$$

and

$$\text{and} \quad \kappa = \left\{2 + [(3+\gamma)/3]^3 (1-\varphi)^{3/\gamma}\right\} [3/(3+\gamma)]^3 (1-\varphi)^{-3/\gamma} \tag{12}$$

respectively, for cylindrical and spherical volumes. Figure 1 shows the dependences of κ calculated by (11) and (12) for $\gamma = \gamma_*$.

2. Basic Equations and the Laws of Conservation That Describe the Dynamics of an Electric Discharge in a Two-Phase Liquid. It is shown above that the presence of a gas content can be allowed for in terms of the polytropic exponents κ. Then for a unit mass of the liquid of a two-phase medium, the equation of state is

$$P = P_1 (\rho/\rho_1)^{\kappa}, \tag{13}$$

where P_1 and ρ_1 are the prescribed pressures and densities.

From the equations of motion, continuity, and energy, for the liquid phase with allowance for its compressibility because of the formation of shock waves in electroexplosion [12], we can determine the velocity, the pressure, and the density as functions of the coordinate and time under the prescribed initial and boundary conditions. It is natural to adopt as boundary conditions the laws of conservation of mass, momentum, and energy on the front of a shock wave that moves with the velocity u_*:

$$\rho_1 (u_1 - u_*) = \rho_2 (u_2 - u_*), \tag{14}$$

$$P_1 + \rho_1 (u_1 - u_*)^2 = P_2 + \rho_2 (u_2 - u_*)^2, \tag{15}$$

$$\varepsilon_1 + \frac{P_1}{\rho_1} + \frac{(u_1 - u_*)^2}{2} = \varepsilon_2 + \frac{P_2}{\rho_2} + \frac{(u_2 - u_*)^2}{2}, \tag{16}$$

$$\varepsilon = \frac{1}{\kappa - 1}\frac{P}{\rho} = \frac{c^2}{\kappa(\kappa - 1)}, \quad c = (\kappa P/\rho)^{1/2}, \tag{17}$$

where c is the velocity of sound. From (14)-(17), u_* is established in terms of the quantities sought. The law of expansion of the boundary of the discharge cavity $a(t)$ is governed by the regime of releasing the power $N(t)$ of the electric energy converted to heat. According to the first law of thermodynamics, the energy-balance equation has the form

$$N(t) = \frac{1}{\kappa - 1}\frac{d(P(a, t)\, V(a, t))}{dt} + P(a, t)\frac{dV(a, t)}{dt}. \tag{18}$$

Formulas (14)-(18) permit a detailed numerical analysis of the problem stated. However investigations showed [1, 2] that the result of modeling depends strongly on the selected form of $N(t)$, hence on the law of expansion of the boundary of the cavity, which have not been established uniquely at present. Specific properties of the problem in question, i.e., thermodynamic properties of a two-phase medium as functions of its fractal structure, are added to this uncertainty. The fractal nature is a property of a strongly nonlinear medium. First of all, it is necessary to establish the form of the function $N(t)$ that allows for electrical properties of the medium as functions of the process itself occurring in it.

3. Rate of Electroexplosion-Energy Release in a Nonlinear Medium. Different model expressions are used to describe the regime of energy release in an electric discharge in a liquid. A "triangular" approximation [1] that describes correctly the time dependence only in the first half-period of the discharge is employed most frequently. The power sinusoidal approximations of [13] fail to allow for the attenuation with time of the power supplied to the discharge. Although the approximations used describe satisfactorily the behavior of $N(t)$ in certain time intervals when the empirical constants are selected appropriately they are not quite convenient for construction of theoretical models because of the inevitability of large errors that occur in analytical operations (differentiation, integration, etc.). The experimental dependence of the power on time is an oscillating and rapidly damping curve [1]. To seek

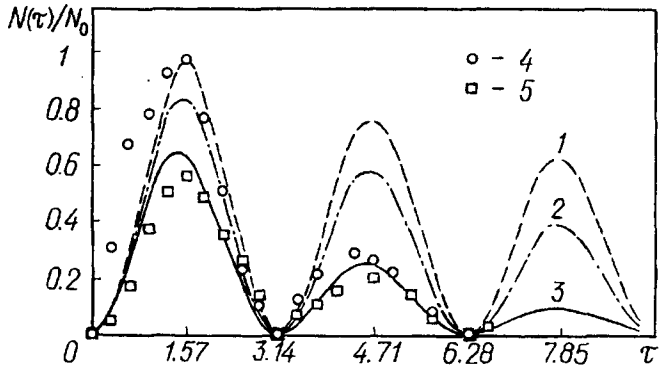

Fig. 2. Released power of the discharge energy vs. τ for: 1) $\alpha_1 = 0.1$, 2) 0.2, 3) 0.3; experimental dependences: 4) according to [1] for $U_0 = 60$ kV, $C = 120\ \mu$F, $L = 1.5\ \mu$G, $l = 0.28$ m; 5) for $U_0 = 25$ kV, $C = 0.4\ \mu$F, $L = 0.9\ \mu$G, $l = 0.018$ m.

the form of $N(t)$ that corresponds to experiment, we allow for the basic specific properties of the phenomenon of a discharge in a liquid medium – its substantially nonlinear character.

The electric circuit of the discharge in the liquid is a nonlinear oscillatory circuit [14] described by the equation

$$L\frac{dI}{dt} + U(I) + \frac{1}{C}\int I dt = 0, \tag{19}$$

where $U(I)$ is the nonlinear voltage, dependent on the current strength I. The volt-ampere characteristic of the electric circuit with nonlinear resistance will be selected in the form

$$U(I) = \alpha_1\left(I + \frac{\beta I^3}{3}\right), \tag{20}$$

where α_1 and β are constants. The choice of the simplest dependence in the form of (20) is substantiated by the fact that oscillatory phenomena must not be described by even powers of the argument, since in averaging they do not lead to the appearance of a constant (nonoscillatory) shift of the function from its equilibrium value. Differentiating (20) with respect to time and introducing a new constant α, we have

$$\frac{d^2I}{d\tau^2} + \alpha(1 + \beta I^2)\frac{dI}{d\tau} + I = 0, \quad \tau = \omega_0 t = \frac{t}{(LC)^{1/2}}, \quad \alpha = \alpha_1 (C/L)^{1/2}. \tag{21}$$

Equation (21) for negative parameters α and β coincides with the Van der Pol equation, known in the theory of nonlinear oscillations [15]. Using the method of variation of parameters, we obtain a solution of (21) in the form

$$I(\tau) = \frac{2\exp(-\alpha\tau/2)\sin\tau}{[4 + \beta - \beta\exp(-\alpha\tau)]^{1/2}}. \tag{22}$$

When $\beta \to 0$ (22) yields the known solution for a linear electric circuit with damping

$$I(\tau) = \exp(-\alpha\tau/2)\sin\tau. \tag{23}$$

The equation sought for the power $N(\tau)$ has the form

$$N(\tau) = IU = \alpha_1 (L/C)^{1/2}[I^2(\tau) + \beta I(\tau)^4/3]. \tag{24}$$

Figure 2 gives time variations in the power for different α_1 ($\beta = \alpha_1^3$), which show that expression (24) describes correctly the experimental dependence of the power on time as an oscillating curve with damping [1]. The latter occurs more rapidly, the larger is α_1.

The commonly used formula for calculating the power

$$N(\tau) = I^2(\tau) R \tag{25}$$

preassumes a linear relationship with a constant coefficient of resistance R between the voltage and the current (Ohm's law). The first term in (24) corresponds to the linear $U = U(I)$, while the second term corresponds to a nonlinear relationship.

4. Dynamics of the Expansion of the Cavity of an Electric Discharge in a Two-Phase Liquid. Using the regularity of the release of electric discharge energy found, we can trace the evolution of the discharge space and the pressure in it. We investigate the effect of the two-phase nature and the attendant fractal nature of the medium on the dynamics of the phenomenon. The indicated characteristics are considered to be integral and referring to the entire volume in which the electric discharge is initiated. For this reason, we will employ only the integral relations (14)-(17) and the laws of conservation of dynamic and thermodynamic (18) quantities without allowance for a space variation in the dynamic characteristics. In the energy-balance equation (18), we use the maximum pressure attained on the shock-wave front. Then from (18) we can seek the time dependence of the boundary of expansion of the discharge cavity $a(t)$, which moves with the shock-wave velocity:

$$u_* = da(t)/dt. \tag{26}$$

From the Hugoniot relations (14)-(17) for a "strong wave" we have [12]

$$P_m = 2\rho_0 u_*^2/(\kappa + 1). \tag{27}$$

We consider that ρ_0 and P_0 are negligibly small ahead of the shock-wave front (in an unperturbed medium). Allowing for formulas (18), (26), and (27), we obtain

$$N(t) = \frac{2\rho_0\kappa}{\kappa^2 - 1}\left(\frac{da(t)}{dt}\right)^2 \frac{dV(a, t)}{dt}, \tag{28}$$

where we disregard the derivative of pressure with respect to time. This approach means that $P(t)$ is determined by a successive approximation: first, P is taken to be P_m; subsequently, $P(t)$ is sought based on the expression found for $a(t)$.

Expressing the volume in terms of the linear dimension of the expansion of the discharge cavity, we write

$$(j + 1) d^j(t)\left(\frac{da(t)}{dt}\right)^3 \frac{2\rho_0\kappa C_j}{\kappa^2 - 1} = N(t), \quad V = C_j d^{j+1}, \quad C_1 = \pi l, \quad C_2 = 4\pi/3. \tag{29}$$

From (29), there follows the law of expansion of the discharge cavity with time

$$a(\tau) = B_j\left(\frac{\kappa^2 - 1}{\kappa}\right)^{1/(3+j)} \left\{\int_0^\tau [N(\tau)]^{1/3} d\tau\right\}^{3/(3+j)} + a(0), \tag{30}$$

where

$$B_j(\tau) = \left(\frac{j + 3}{3}\right)^{3/(3+j)} \left[\frac{1}{2\rho_0 (1 + j) C_j}\right]^{1/(j+3)}.$$

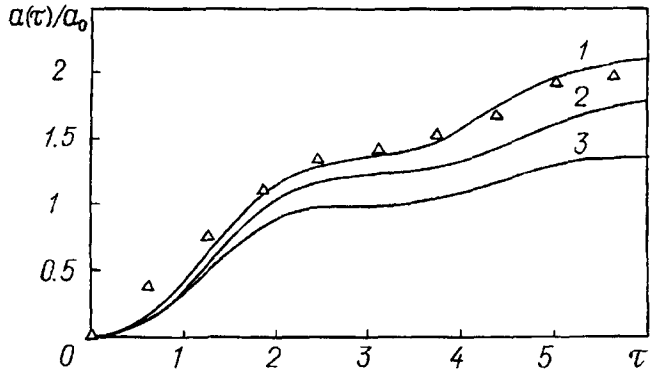

Fig. 3. Expansion of the cavity of a discharge vs. time for the case of cylindrical symmetry (for α_1, see 1-3 in Fig. 2); experimental data of high-speed photorecording according to [1] for $U_0 = 43.5$ kV, $C = 6 \cdot 10^{-6}$ F.

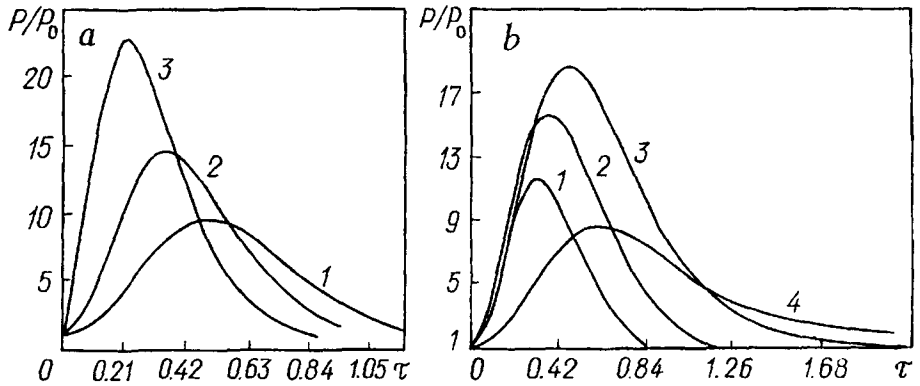

Fig. 4. Pressure in the discharge channel vs. time for the case of cylindrical symmetry: a) $\kappa = 2.5$; α_1: 1) 0.1, 2) 0.4, 3) 0.9; b) $\alpha_1 = 0.4$; κ: 1) 3, 2) 2.27, 3) 1.63, 4) 1.11.

In particular,

$$B_1 = (LC)^{3/8} (16/27C_1\rho_0)^{1/4}, \quad B_2 = (LC)^{3/10} (125/162C_2\rho_0)^{1/5}.$$

Calculations by (30) (see Fig. 3) show that this expression describes correctly the character of the expansion with time of the increase in $a(\tau)$ observed in the experiment [16]. The effect of the two-phase nature of the medium on the dynamics of a discharge channel is allowed for in terms of the polytropic exponent κ.

The evolution of the pressure with time in an expanding cavity of the discharge can also be traced using (18), considering now $P = P(t)$ to be a variable and employing the expression found for $a(\tau)$. When the volume is expressed in terms of $a(\tau)$ Eq. (18) will be represented as

$$dP(\tau)/d\tau + f(\tau) P(\tau) = g(\tau), \tag{31}$$

$$f(\tau) = \frac{\kappa(j+1)}{a(\tau)^{1-j}} \frac{da(\tau)}{d\tau}, \tag{32}$$

$$g(\tau) = (\kappa - 1) N(\tau) (LC)^{1/2} C_j a(\tau)^{1-j}. \tag{33}$$

Solution of Eqs. (31)-(33) has the form

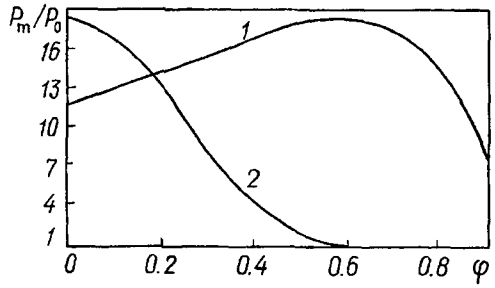

Fig. 5. Maximum pressure vs. gas content of a flow by the averaged space (1) and local (2) fractal scales.

$$P(\tau) = \exp\left(-\int f(\tau)\, d\tau\right)\left\{1 + \int\left[g(\tau)\exp\left(+\int f(\tau)\, d\tau\right)\right] d\tau\right\}. \tag{34}$$

The pressures calculated by formula (34) and given in Fig. 4 confirm the nonmonotonic behavior of $P(\tau)$ observed in the experiment [1]. Although $a(\tau)$ increases and decreases smoothly (both theoretically and experimentally), $P(\tau)$ is ambigously determined by the inverse proportionality to the area of the discharge cavity ($\sim a^2(\tau)$) since its evolution depends on the derivative $a(\tau)$. Figure 4 shows the time variation of the pressure in the discharge channel as a function of the parameter α_1 (the linear portion of electrical resistance) and the polytropic exponent k. The maximum pressure P_m increases with α_1; the time interval in which it is attained decreases. The effect of the polytropic exponent (the gas content φ) on the pressure evolution is unambiguous. In a closed volume, where the average gas content is determined, the pressure can be the highest for $\varphi \sim 0.6$ (Fig. 5), which corresponds to $\kappa \sim 1.4$, which describes adiabatic expansion.

In the case of comparability of the interelectrode gap l with the dimensions of fractal structures r_m, which occurs when $l \sim r_m \sim 1$ mm, we need to employ the local gas content φ. The polytropic exponent κ increases sharply with φ, the process becoming isochoric. As the gas content φ increases the maximum attainable pressure drops (Fig. 5), and the evolution of the pressure with time becomes steeper. However, here, it should be taken into account that, for $\varphi \geq 0.3$ ($\kappa \geq 3$), the electrohydraulic discharge is practically not initiated.

NOTATION

V, volume of the medium; $V(\gamma)$, fractal volume of the liquid; L, inductance; C, capacity; Re and $\mathrm{Re_{cr}}$, Reynolds number and its critical value; k, Boltzmann constant; a, characteristic scale of the fractal structure; a_{eff}, effective scale; l, interelectrode gap (the length of the discharge of cylindrical symmetry); N, power of energy release in electric discharge in the liquid; ω_0, cyclic frequency dependent on electrical characteristics; t, time; τ, cyclic time; $j = 1, 2$, for cylindrical and spherical geometries of the discharge cavity. Subscripts: 0 and m denote the minima and the maxima of the quantities; r, relaxation; t, thermal.

REFERENCES

1. E. V. Krivitskii, *Dynamics of Electroexplosion in a Liquid* [in Russian], Kiev (1986).
2. V. V. Shamko and V. V. Kucherenko, *Electrophysical and Hydrodynamic Processes of an Electric Discharge in Condensed Media* [in Russian], Kiev (1987), pp. 84-91.
3. V. V. Ivanov, O. V. Matvienko, V. N. Taruzhin, and O. M. Rybka, *Processes of Energy Conversion in Electroexplosion* [in Russian], Kiev (1988), pp. 58-66.
4. A. M. Beskaravainyi, V. G. Kovalev, and V. A. Pozdeev, *Basic Problems of Discharge-Pulse Technology* [in Russian], Kiev (1980), pp. 127-134
5. L. D. Landau and E. M. Livshits, *Statistical Physics*, Part 1 [in Russian], Moscow (1979).
6. L. D. Landau and E. M. Livshits, *Hydrodynamics* [in Russian], Moscow(1986).
7. Yu. L. Klimontovich, *Statistical Physics* [in Russian], Moscow (1982).
8. E. Feder, *Fractals* [Russian translation], Moscow (1991).

9. K. Kusaiynov, "*Fractal Theory of the Motion of a Two-Phase Liquid in a Tube,*" Deposited in KazGosINTI, No. 7204, Ka96.
10. V. N. Shtern (ed. M. A. Gol'dshtik), *Structural Turbulence* [in Russian], Novosibirsk (1982), pp. 136-161.
11. Z. Zh. Zhanabaev, *Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekhn. Nauk*, Issue 4, No. 15, 57-60 (1988).
12. B. L. Rozhdestvenskii and N. N. Yanenko, *Systems of Quasilinear Equations* [in Russian], Moscow (1978).
13. V. A. Pozdeev and Zh. A. Ishchenko, *Electric-Discharge Processes: Theory, Experiment, and Practice* [in Russian], Kiev (1984), pp. 58-64.
14. P. M. Kolesnikov, *Introduction to Nonlinear Electrodynamics* [in Russian], Minsk (1971).
15. N. N. Bogolyubov and Yu. A. Mitropol'skii, *Asymptotic Methods in the Theory of Nonlinear Oscillations* [in Russian], Moscow (1974).
16. V. V. Kucherenko and V. V. Shamko, *Processes of Energy Conversion in Electroexplosion* [in Russian], Kiev (1988), pp. 90-94.